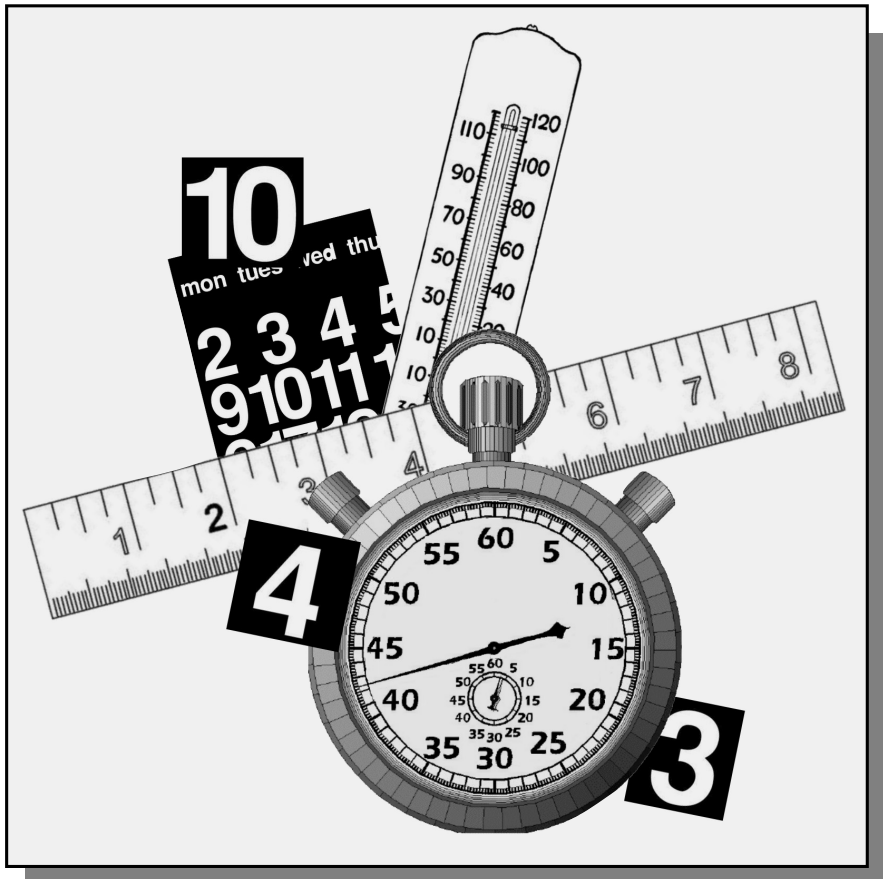
Intermediate Math Skills



Chapter 6

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"The study of Mathematics cannot be replaced by any other activity that will train and develop man's purely logical faculties to the same level of rationality."

—C. O. Oakley



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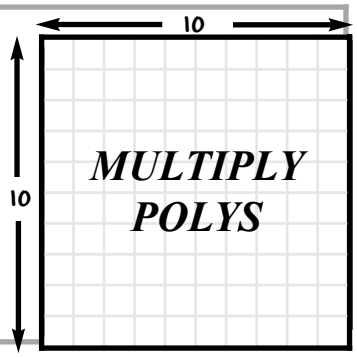
Intermediate Math Skills



Algebra

Lesson 1

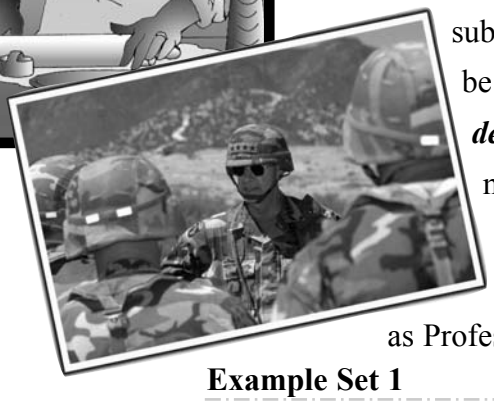
Chapter 6



Section 1

Lessons 1-5





Problems in life are often multiplied because of our ***attitudes*** more than anything else. Remember, Professor K believes that problems are actually opportunities in disguise. When facing a problem, you have the opportunity to succeed. Professor K knows that you have the opportunity to succeed at math.

Polynomials not only can be added and subtracted, but polynomials can also be multiplied. Professor K will ***demonstrate*** in this lesson how to multiply monomials times monomials, binomials and trinomials. Look at **Example Set 1** as Professor K multiplies two monomials.

Example Set 1

Multiply the following monomials.

a) $3x(4x) =$
$(3 \bullet 4)(x \bullet x)$
$12x^2$

a) Professor K multiplies these two monomials by using the Commutative Property to rearrange the numbers and variables. Professor K then performs the multiplications: $3 \bullet 4 = 12$ and $x \bullet x = x^1 \bullet x^1 = x^{1+1} = x^2$ to obtain the product $12x^2$. Remember, to multiply variables you must add the exponents.

VOCABULARY

Attitudes: ways of thinking or behaving
Demonstrate: to show how something works

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b) $2x(15y)(3x^2)$
$(2 \bullet 15 \bullet 3)$
$(x \bullet x^2)(y) =$
$90x^3y$

b) Professor K multiplies these three monomials by using the Commutative Property to rearrange the numbers and variables. Professor K then performs the multiplications: $2 \bullet 15 \bullet 3 = 90$ and $x \bullet x^2 = x^1 \bullet x^2 = x^{1+2} = x^3$ to obtain the product $90x^3y$. Remember, to multiply variables you must add the exponents.

Example Set 2 shows how Professor K multiplies a monomial times a binomial.

Example Set 2

Multiply the following polynomials.

a) $6x(5x - 7)$
$6x \bullet 5x - 6x \bullet 7$
$(6 \bullet 5)(x \bullet x) - (6 \bullet 7)x$
$30x^2 - 42x$

a) Professor K multiplies the monomial, $6x$, times the binomial, $5x - 7$. Professor K distributes the multiplier, $6x$, by multiplying each term of the binomial within the parentheses. Professor K performs the multiplications and determines the solution to be $30x^2 - 42x$.

b) $2x(2x + 3)$
$2x \bullet 2x + 2x \bullet 3$
$(2 \bullet 2)(x \bullet x) + (2 \bullet 3)x$
$4x^2 + 6x$

b) Professor K multiplies the monomial, $2x$, times the binomial, $2x + 3$. Professor K distributes the multiplier, $2x$, by multiplying each term of the binomial within the parentheses. Professor K performs the multiplications and determines the solution to be $4x^2 + 6x$.

c) $5x(2x + 2)$
$5x \bullet 2x + 5x \bullet 2$
$(5 \bullet 2)(x \bullet x) + (2 \bullet 5)x$
$10x^2 + 10x$

c) Professor K multiplies the monomial, $5x$, times the binomial, $2x + 2$. Professor K distributes the multiplier, $5x$, by multiplying each term of the binomial within the parentheses. Professor K performs the multiplications and determines the solution to be $10x^2 + 10x$.

Notice that in multiplying a monomial times a binomial, the Distributive Property is used to determine the solution. In **Example Set 3**, Professor K shows you how to multiply a monomial by a trinomial.

Example Set 3

Multiply the following polynomials.
$2x(5a + 12b + 8c)$

Step 1 $2x \bullet 5a + 2x \bullet 12b + 2x \bullet 8c$

Professor K begins to determine the product of the monomial and trinomial by distributing the $2x$ within the parentheses. (See Step 1.)

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Step 2	$(2 \cdot 5)(x \cdot a) + (2 \cdot 12)(x \cdot b) + (2 \cdot 8)(x \cdot c)$	In Step 2, Professor K rearranges the factors to accomplish multiplication. Remember, Professor K can rearrange terms because of the Commutative Property, which states that $a \cdot b = b \cdot a$.
Step 3	$10ax + 24bx + 16cx$	In Step 3, Professor K completes the multiplications and writes the products, $10ax + 24bx + 16cx$.

You can multiply various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to multiply polynomials by working the activities for this lesson.

LIFE PRINCIPLE



"By asking for the impossible we obtain the possible."

—Italian Proverb

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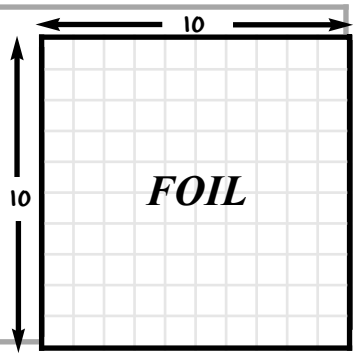
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Lesson 2

Algebra

Chapter 6



Section 1

Lessons 1-5







Professor K

You have probably purchased a new item that required ***assembly***. If you are like Professor K, you also probably attempted to assemble the item without reading the directions. When Professor K does this, he usually makes mistakes and has to read the instructions and start over. This process takes twice as long as it would have if Professor K had read and followed the instructions in the first place. Certain procedures should be accomplished first and certain procedures should be accomplished last.

In mathematics, certain procedures and operations must be accomplished first, second, third, …. and last. You studied such a process when you learned the Order of Operations, Please Excuse My Dear Aunt Sally (PEMDAS) in Chapter 2, Lesson 14.

In this lesson, Professor K will introduce another important procedure that orders the way you multiply polynomials. This method is called **F**irst, **O**uter, **I**nner, **L**ast, ***FOIL***. This acronym means that the procedure for multiplying polynomials is to multiply the *first* terms of each polynomial, then the *outer* terms, then the *inner* terms and finally, the *last* terms. Look at **Example Set 1** as Professor K demonstrates how to FOIL the multiplication of two binomials with the same operation symbol, addition. Follow the steps closely so that you too can use FOIL to determine the product of two binomials.

VOCABULARY

Assembly: to bring or to put together

FOIL: First, Outer, Inner, Last; a method for multiplying binomials

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Example Set 1

Multiply the following binomials.

$(2x + 1)(3x + 2)$



First	$(2x + 1)(3x + 2)$ $2x \bullet 3x = (2 \bullet 3)(x \bullet x) = 6x^2$	Professor K multiplies the **F**irst terms of the two binomials: $2x \bullet 3x = 6x^2$.
Outer	$(2x + 1)(3x + 2)$ $2x \bullet 2 = (2 \bullet 2)x = 4x$	Professor K multiplies the **O**uter terms of the two binomials: $2x \bullet 2 = 4x$.
Inner	$(2x + 1)(3x + 2)$ $1 \bullet 3x = 3x$	Professor K multiplies the **I**nner terms of the two binomials: $1 \bullet 3x = 3x$.
Last	$(2x + 1)(3x + 2)$ $1 \bullet 2 = 2$	Professor K multiplies the **L**ast terms of the two binomials: $1 \bullet 2 = 2$.
	F O I L $6x^2 + 4x + 3x + 2$ $6x^2 + 7x + 2$	Professor K now places the products of FOIL in order as shown. Professor K then simplifies the expression, combining the like terms $4x$ and $3x$. $(3 + 4)x = 7x$. Professor K then determines the product of the two binomials: $(2x + 1)(3x + 2) = 6x^2 + 7x + 2$.

Example Set 2 shows how Professor K multiplies another binomial times a binomial. Both binomials contain the same operation symbol, *subtraction*.

Example Set 2

Multiply the following binomials.

$(6x - 1)(5x - 7)$

First	$(6x - 1)(5x - 7)$ $6x \bullet 5x = (6 \bullet 5)(x \bullet x) = 30x^2$	Professor K multiplies the **F**irst terms of the two binomials: $6x \bullet 5x = (6 \bullet 5)(x^1 \bullet x^1) = 30x^{1+1} = 30x^2$.
Outer	$(6x - 1)(5x - 7)$ $6x \bullet -7 = (6 \bullet -7)(x) = -42x$	Professor K multiplies the **O**uter terms of the two binomials: $6x \bullet -7 = -42x$.

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Inner	$(6x - 1)(5x - 7)$ $-1 \bullet 5x = (-1 \bullet 5)(x) = -5x$	Professor K multiplies the **I**nner terms of the two binomials: $-1 \bullet 5x = -5x$.
Last	$(6x - 1)(5x - 7)$ $-1 \bullet -7 = (-1 \bullet -7) = 7$	Professor K multiplies the **L**ast terms of the two binomials: $-1 \bullet -7 = 7$. Remember, negative times negative = positive.
	$30x^2 - 42x - 5x + 7$ $30x^2 - 47x + 7$	Professor K now places the products of FOIL in order as shown. Professor K then simplifies the expression, combining the like terms: $-42x$ and $-5x$. $(-42 + -5)x = -47x$. Finally, Professor K determines the product of the two binomials: $(6x - 1)(5x - 7) = 30x^2 - 47x + 7$.

In **Example Set 3**, Professor K shows you how to multiply a binomial by a binomial. The two binomials have different operation symbols; one addition, the other subtraction.



Example Set 3

Multiply the following binomials.

$(3x - 1)(3x + 4)$

First	$(3x - 1)(3x + 4)$ $3x \bullet 3x = (3 \bullet 3)(x \bullet x) = 9x^2$	Professor K multiplies the **F**irst terms of the two binomials: $3x \bullet 3x = (3 \bullet 3)(x^1 \bullet x^1) = 9x^{1+1} = 9x^2$.
Outer	$(3x - 1)(3x + 4)$ $3x \bullet 4 = (3 \bullet 4)(x) = 12x$	Professor K multiplies the **O**uter terms of the two binomials: $3x \bullet 4 = 12x$.
Inner	$(3x - 1)(3x + 4)$ $-1 \bullet 3x = (-1 \bullet 3)(x) = -3x$	Professor K multiplies the **I**nner terms of the two binomials: $-1 \bullet 3x = -3x$.
Last	$(3x - 1)(3x + 4)$ $-1 \bullet 4 = (-1 \bullet 4) = -4$	Professor K multiplies the **L**ast terms of the two binomials: $-1 \bullet 4 = -4$. Remember, negative times positive = negative.

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$9x^2 + 12x - 3x - 4$
$9x^2 + (12 - 3)x - 4$
$9x^2 + 9x - 4$

Professor K now places the products of FOIL in order as shown. Professor K then simplifies the expression, combining the like terms $12x$ and $-3x$: $(12 + -3)x = 9x$. Finally, Professor K determines the product of the two binomials; $(3x - 1)(3x + 4) = 9x^2 + 9x - 4$.

You can multiply various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to multiply polynomials by working the activities for this lesson.

Life Principle



"As thou valuest thyself,
others will esteem thee."

—Gaelic Proverb

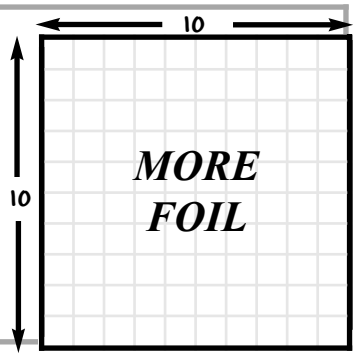
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Lesson 3

Algebra

Chapter 6



Section 1

Lessons 1-5





In business and the military, a person is ***promoted*** to positions of greater ***responsibility*** after he has proven himself with lesser responsibilities or tasks. In life, you must be faithful to take care of little things, so that you can be given bigger things.

In this chapter you have worked your way through monomial multiplications of binomials and trinomials, and multiplications of two binomials. This lesson will take you one more step into the world of mathematics by showing you how to multiply a binomial by a trinomial.

Look at **Example Set 1** as Professor K demonstrates how FOIL, the multiplication of two binomials, is really a form of the Distributive Property. Professor K will use this fact to show you in further examples how to multiply a binomial by a trinomial using the Distributive Property.

Example Set 1

Multiply the following binomials.

$(2x + 1)(3x + 2)$

Step 1	$2x(3x + 2) + 1(3x + 2)$	In Step 1, Professor K distributes each term of the first binomial to the second binomial.
Step 2	$[2x(3x) + 2x(2)] + [1(3x) + 1(2)]$	In Step 2, Professor K distributes the terms in front of the binomial into the parentheses.

VOCABULARY

Promoted: raised to a more important job
Responsibility: having charge and being accountable for certain duties

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Step 3	$[(2 \bullet 3)(x \bullet x) + (2 \bullet 2)x] + [(1 \bullet 3)x + (1 \bullet 2)]$	In Step 3, Professor K arranges the numbers and variables within the square brackets for multiplication.
Step 4	$6x^2 + 4x + 3x + 2$	In Step 4, Professor K performs the multiplications within the parentheses contained within the square brackets.
Step 5	$6x^2 + (4 + 3)x + 2 = 6x^2 + 7x + 2$	In Step 5, Professor K adds like terms to determine the product of the two binomials, $(2x + 1)$ and $(3x + 2)$ to be $6x^2 + 7x + 2$. Notice that this is the same product that Professor K obtained in **Example Set 1** of the previous lesson using the FOIL method.

Example Set 2 shows how Professor K multiplies a binomial times a trinomial using the Distributive Property as he did in **Example Set 1**.

Example Set 2

Multiply the following polynomials.

$(2x - 5)(2a - b + 4c)$



Step 1	$2x(2a - b + 4c)$ $-5(2a - b + 4c)$	In Step 1, Professor K distributes each term of the binomial to the trinomial.
Step 2	$[2x(2a) - 2x(b) + 2x(4c)] - [5(2a) - 5(b) + 5(4c)]$	In Step 2, Professor K distributes the terms in front of the binomial into the parentheses. Notice that positive five is distributed within the second algebraic expression. The negative sign remains on the outside of the bracket for multiplication later.
Step 3	$[(2 \bullet 2)(x \bullet a) - 2(x \bullet b) + (2 \bullet 4)(x \bullet c)] - [(5 \bullet 2)a - 5b + (5 \bullet 4)c]$	In Step 3, Professor K arranges the numbers and variables within the square brackets for multiplication.
Step 4	$[4xa - 2xb + 8xc] - [10a - 5b + 20c]$	In Step 4, Professor K performs the multiplications within the parentheses contained within the square brackets.

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Step 5	$4xa - 2xb + 8xc - 10a + 5b - 20c$	In Step 5, Professor K removes the square brackets and distributes the -1 that is multiplied into [$10a - 5b + 20c$]. This changes each of the signs within the square brackets to: $- 10a + 5b - 20c$.

By following the steps in these two examples you will be able to multiply other polynomials of multiple terms together. Show Professor K that you understand how to multiply polynomials by working the activities for this lesson.

LIFE PRINCIPLE



"Be the first to the field and the last to the couch."

—Chinese Proverb

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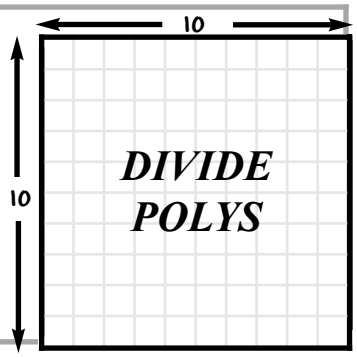
Intermediate Math Skills



Algebra

Lesson 4

Chapter 6



Section 1

Lessons 1-5







Many large problems are solved by dividing them into smaller problems. Division of polynomials can be ***accomplished*** by writing the division problem as a ***fraction***. The larger fraction is then divided into smaller fractions with like denominators. These fractions are then reduced as you did in previous lessons.

In this chapter, you have learned how to multiply polynomials. This lesson will show you how to accomplish the opposite operation, *division*. Professor K will use the reduction of fractions to accomplish this.

Look at **Example Set 1** as Professor K demonstrates how to use fractions to divide polynomials. In previous lessons you changed fractions into division problems. Professor K will now change division problems into fractions to determine the solutions.

Example Set 1

Complete the following division problem.

$(2x^2 + 2x) \div 2x$

Step 1 $\dfrac{2x^2 + 2x}{2x}$

In Step 1, Professor K changes the division problem into a fraction with the divisor, $2x$, placed in the denominator.

Step 2 $\dfrac{2x^2}{2x} + \dfrac{2x}{2x}$

In Step 2, Professor K separates the fraction into smaller fractions with like denominators.

VOCABULARY

Accomplished: succeeded in doing

Fraction: a method of comparing two numbers; another way to show division

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Step 3 $\frac{2x \bullet x}{2x} + \frac{2x}{2x}$

In Step 3, Professor K separates $2x^2$ into its factors, $2x$ and x.

Step 4 $\frac{\cancel{2}\,\cancel{x} \bullet x}{\cancel{2}\,\cancel{x}} + \frac{\cancel{2}\,\cancel{x}}{\cancel{2}\,\cancel{x}}$

In Step 4, Professor K reduces the fractions as shown; $^2/_2 = 1$, $^x/_x = 1$, and $^{2x}/_{2x} = 1$.

Step 5 $1 \bullet 1 \bullet x + 1 = x + 1$

In Step 5, Professor K multiplies the results of Step 4 and determines the solution to $(2x^2 + 2x) \div 2x = x + 1$.

Example Set 2 shows how Professor K divides a trinomial by a monomial using the reduction of fractions as he did in **Example Set 1**.

Example Set 2

Complete the following division problem.

$(24x^3 - 40x^2 + 16x) \div (4x)$

Step 1 $\frac{24x^3 - 40x^2 + 16x}{4x}$

In Step 1, Professor K changes the division problem into a fraction with the divisor, $4x$, placed in the denominator.

Step 2 $\frac{24x^3}{4x} - \frac{40x^2}{4x} + \frac{16x}{4x}$

In Step 2, Professor K separates the fraction into smaller fractions with like denominators.

Step 3 $\frac{6 \bullet 4 \bullet x \bullet x^2}{4 \bullet x} - \frac{10 \bullet 4 \bullet x \bullet x}{4 \bullet x} + \frac{16 \bullet x}{4 \bullet x}$

In Step 3, Professor K separates the numerators and denominators of the separated fractions into factors as shown.

Step 4 $\frac{6 \bullet \cancel{4} \bullet \cancel{x} \bullet x^2}{\cancel{4} \bullet \cancel{x}} - \frac{10 \bullet \cancel{4} \bullet \cancel{x} \bullet x}{\cancel{4} \bullet \cancel{x}} + \frac{4 \bullet \cancel{4} \bullet \cancel{x}}{\cancel{4} \bullet \cancel{x}}$

In Step 4, Professor K reduces the fractions as shown; $^4/_4 = 1$ and $^x/_x = 1$.

Step 5 $6x^2 - 10x + 4$

In Step 5, Professor K multiplies the results of Step 4 and determines the solution to be $6x^2 - 10x + 4$.

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By following the steps in these two examples you will be able to divide other polynomials of multiple terms. Show Professor K that you understand how to divide polynomials by working the activities for this lesson.

Life Principle



"Laziness may appear attractive, but work gives satisfaction."

—Anne Frank (1929-1945)

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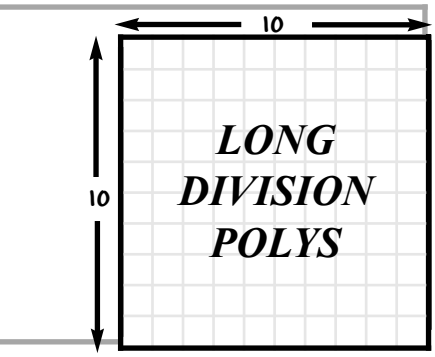
Algebra

Lesson 5

Chapter 6

Section 1

Lessons 1-5









Most cultures have ***traditions*** that ***dictate*** how or when things are done. Many families have traditional foods that are eaten at Christmas, Thanksgiving or the Fourth of July. Some families always go to Grandma's house for these holidays. Traditions are good because they help keep families and nations together. Traditions also remind us of our ***heritage***.

Many problems can be solved in traditional ways. In the previous lesson, Professor K showed you how to divide by changing a division problem containing polynomials into fractions and then reducing the fractions. Polynomials can also be divided in the traditional way by using long division.

Professor K will demonstrate to you in this lesson how to divide polynomials using long division.

Look at **Example Set 1** as Professor K reviews the long division for you using only numbers and no variables.

VOCABULARY

Tradition: a time-honored practice passed down from generation to generation
Dictate: to say or command
Heritage: something inherited and passed down from preceding generations; a tradition

Example Set 1

Divide the following using long division.

$246 \div 2$

Step 1 $2\overline{)246}$

In Step 1, Professor K places the divisor, 2 (the number of equal groups), in front of the tabletop. Professor K places the dividend, 246 (the number being divided into equal parts), under the tabletop as shown.

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Step 2

$$\begin{array}{r} 1__ \\ 2\overline{)246} \\ \underline{-2}\downarrow \\ 04 \end{array}$$

In Step 2, Professor K determines that the divisor, 2, divides the number 2 in the hundreds place of the dividend, 246, one time. Professor K places a 1 in the quotient row above the tabletop. Professor K then multiplies (1 × 2), subtracts (2 - 2) and brings down the number 4 from the tens place of the dividend as shown.

Step 3

$$\begin{array}{r} 12_ \\ 2\overline{)246} \\ \underline{-2}\downarrow \\ 04\downarrow \\ \underline{-4} \\ 06 \end{array}$$

In Step 3, Professor K repeats the process described in Step 2, determining that the divisor (2) divides the 4 in the dividend twice. Professor K places the 2 in the quotient row, multiplies (2 × 2 = 4), then subtracts (4 - 4 = 0), and brings down the 6 from the ones place of the dividend as shown.

Step 4

$$\begin{array}{r} 123 \\ 2\overline{)246} \\ \underline{-2}\downarrow \\ 04\downarrow \\ \underline{-4} \\ 06 \\ \underline{-6} \\ 0 \end{array}$$

In Step 4, Professor K repeats the process described in Steps 2 and 3, determining that 2 divides 6 three times, placing a 3 in the quotient row, multiplying and subtracting (6 - 6 = 0). Professor K has determined that the quotient (answer) of 246 ÷ 2 is 123.

In **Example Set 2**, Professor K will show you how to divide a polynomial by a monomial.

Example Set 2

Complete the following division problem containing a polynomial.

$(2x + 4) \div 2$

Step 1

$$2\overline{)2x+4}$$

In Step 1, Professor K places the divisor, 2 (the number of equal groups), in front of the tabletop. Professor K places the dividend, $2x + 4$ (the polynomial being divided into equal parts), under the tabletop as shown.

Step 2

$$\begin{array}{r} x___ \\ 2\overline{)2x+4} \\ \underline{-2x}\quad\downarrow \\ 0\quad +4 \end{array}$$

In Step 2, Professor K looks at the first term, $2x$, of the polynomial under the table top and determines that the divisor, 2, divides the term, $2x$, x number of times ($^{2x}/_{2} = x$). Professor K places an x in the quotient row above the $2x$ of the dividend. Professor K then multiplies, subtracts and brings down the next term, the number 4.

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Step 3

$$2\overline{)2x+4}\quad x+2$$
$$-2x$$
$$0+4$$
$$-4$$
$$0$$

In Step 3, Professor K repeats the process described in Step 2, determining that 2 divides 4 twice, and placing a 2 in the quotient row above the 4. Professor K then multiplies ($2 \times 2 = 4$) and subtracts ($4 - 4 = 0$). Professor K has determined through long division that the quotient of $(2x + 4) \div 2$ is $x + 2$.

Example Set 3 shows how Professor K divides a trinomial by a monomial using long division as he did in **Example Set 2**. Notice that this is the same problem Professor K solved with fractions in **Example Set 2** in the previous lesson.

Example Set 3

Complete the following division problem.

$(24x^3 - 40x^2 + 16x) \div (4x)$

Step 1

$$4x\overline{)24x^3-40x^2+16x}$$

In Step 1, Professor K places the divisor, $4x$ (the number of equal groups), in front of the tabletop. Professor K places the dividend, $24x^3 - 40x^2 + 16x$, (the polynomial being divided into equal parts), under the tabletop as shown.

Step 2

$$4x\overline{)24x^3-40x^2+16x}\quad 6x^2$$
$$-24x^3$$
$$0\quad -40x^2$$

In Step 2, Professor K looks at the first term ($24x^3$) of the polynomial under the table top and determines that the divisor ($4x$) divides the term, $24x^3$, $6x^2$ number of times ($^{24x^3}/_{4x} = 6x^2$). Professor K places $6x^2$ in the quotient row above the $24x^3$ of the dividend. Professor K then multiplies, subtracts and brings down the next term ($-40x^2$).

Step 3

$$4x\overline{)24x^3-40x^2+16x}\quad 6x^2-10x$$
$$-24x^3\qquad -40x^2$$
$$0\qquad +40x^2$$
$$0\quad +16x$$

In Step 3, Professor K repeats the process described in Step 2, determining that $4x \bullet -10x = -40x^2$, placing a $-10x$ in the quotient row, multiplying, subtracting ($-40x - (-40x) = -40x + 40x = 0$) and bringing down the $16x$.



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Step 4

$$
\begin{array}{r}
6x^2 - 10x + 4 \\
4x\overline{)\,24x^3 - 40x^2 + 16x} \\
\underline{-24x^3} \\
0 \quad -40x^2 \\
\underline{+40x^2} \\
0 \quad +16x \\
\underline{-16x} \\
0
\end{array}
$$

In Step 4, Professor K repeats the process described in Steps 2 and 3, determining that $4x \bullet 4 = 16x$. Professor K places a 4 in the quotient row, multiplying, subtracting ($16x - 16x = 0$). Professor K has determined through long division that the quotient of ($24x^3 - 40x^2 + 16x$) and ($4x$) is $6x^2 - 10x + 4$.

By following the steps in these three examples, you will be able to divide other polynomials of multiple terms by a monomial. Show Professor K that you understand how to divide polynomials by a monomial by working the activities for this lesson.

LIFE PRINCIPLE



"He who goes the wrong road must go the journey twice over."

—Costa Rican Proverb

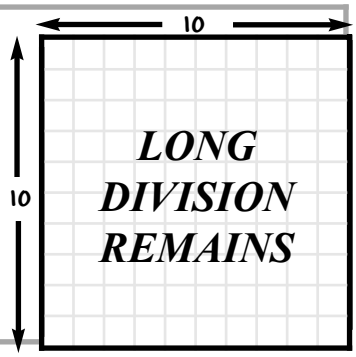
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Lesson 6

Algebra

Chapter 6

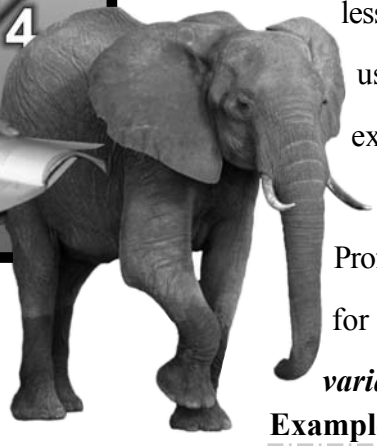


Section 2

Lessons 6-10



In Lesson 5, Professor K ***demonstrated*** how to determine the quotient of a polynomial and a monomial using long division. In Lesson 5, the divisors divided the dividends evenly, thus quotients did not contain a remainder. In this lesson, Professor K will demonstrate the use of long division and how to express a ***remainder*** in the final answer. Look at **Example Set 1** as Professor K reviews the long division for you using only numbers and no ***variables***.



VOCABULARY

Demonstrated: showed or explained
Remainder: what is left over in a division problem
Variables: a letter used to represent a number

Example Set 1

Divide the following equation using long division.

$245 \div 2$

Step 1 $2\overline{)245}$

In Step 1, Professor K places the divisor, 2 (the number of equal groups), in front of the tabletop. Professor K places the dividend, 245 (the number being divided into equal parts), under the tabletop as shown.

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Step 2

$$\begin{array}{r} 1_ \\ 2\overline{)245} \\ \underline{-2\downarrow} \\ 04 \end{array}$$

In Step 2, Professor K determines that the divisor, 2, divides the number 2 in the hundreds place of the dividend, 245, one time. Professor K places a 1 in the quotient row above the 2 of the dividend. Professor K then multiplies, subtracts and brings down the number 4 from the tens place of the dividend as shown.

Step 3

$$\begin{array}{r} 12_ \\ 2\overline{)245} \\ \underline{-2\downarrow\ \ } \\ 04\ \ \\ \underline{-4\downarrow} \\ 05 \end{array}$$

In Step 3, Professor K repeats the process described in Step 2, determining that $2 \times 2 = 4$, placing a 2 in the quotient row, multiplying, subtracting ($4 - 4 = 0$), and bringing down the 5 from the ones place of the dividend as shown.

Step 4

$$\begin{array}{r} 122 \\ 2\overline{)245} \\ \underline{-2\downarrow\ \ } \\ 04\ \ \\ \underline{-4\downarrow} \\ 05 \\ \underline{-4} \\ 1 \end{array}$$

In Step 4, Professor K repeats the process described in Steps 2 and 3, determining that $2 \times 2 = 4$, which is one less than 5. Professor K places a 2 in the quotient row, multiplies and subtracts ($5 - 4 = 1$). Professor K determines that the quotient of $245 \div 2$ is 122 with a remainder of 1. Professor K expresses the quotient (whole number) with a remainder (fraction) which consists of the remainder being placed in the numerator and the divisor being placed in the denominator. Thus, $245 \div 2 = 122\frac{1}{2}$.

In **Example Set 2**, Professor K will show you how to divide a polynomial by a monomial when the quotient contains a remainder.

Example Set 2

Complete the following division problem containing a polynomial.

$(2x + 7) \div 2$

Step 1

$$2\overline{)2x+7}$$

In Step 1, Professor K places the divisor, 2 (the number of equal groups), in front of the tabletop. Professor K places the dividend, $2x + 7$ (the polynomial being divided into equal parts), under the tabletop as shown.

Step 2

$$\begin{array}{r} x__ \\ 2\overline{)2x+7} \\ \underline{-2x\ \ \downarrow} \\ 0+7 \end{array}$$

In Step 2, Professor K looks at the first term, $2x$, of the polynomial under the table top and determines that the divisor, 2, divides the term, $2x$, x number of times ($\frac{2x}{2} = x$). Professor K places an x in the quotient row above the $2x$ of the dividend. Professor K then multiplies, subtracts and brings down the next term, the number 7.

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Step 3

$$2\overline{)2x+7}\quad x+3$$
$$-2x$$
$$0+7$$
$$-6$$
$$1$$

In Step 3, Professor K repeats the process described in Step 2, determining that 3 × 2 = 6, placing a 3 in the quotient row, multiplying and subtracting (7 - 6 = 1). Professor K has determined through long division that the quotient of $(2x + 7) \div 2$ is $x + 3$ with a remainder of 1. Professor K expresses the quotient (whole number) with a remainder (fraction) which consists of the remainder (1) being placed in the numerator and the divisor (2) being placed in the denominator. $(2x + 7)$ 2 is $x + 3 + {}^{1}/_{2}$ or $x + 3{}^{1}/_{2}$.

Example Set 3 shows how Professor K divides a trinomial by a monomial using long division as he did in **Example Set 2**. This quotient will also contain a remainder.

Example Set 3

Complete the following division problem.

$(24x^2 - 40x + 16) \div (4x)$

Step 1

$$4x\overline{)24x^2-40x+16}$$

In Step 1, Professor K places the divisor, $4x$ (the number of equal groups), in front of the tabletop. Professor K places the dividend, $24x^2 - 40x + 16$ (the polynomial being divided into equal parts), under the tabletop as shown.

Step 2

$$4x\overline{)24x^2-40x+16}\quad 6x_____$$
$$-24x^2$$
$$0\ -40x$$

In Step 2, Professor K looks at the first term ($24x^2$) of the polynomial under the tabletop and determines that the divisor ($4x$) divides the term, $24x^2$, $6x$ times (${}^{24x^2}/_{4x} = 6x$). Professor K places $6x$ in the quotient row above the $24x^2$ of the dividend. Professor K then multiplies, subtracts and brings down the next term ($-40x$).

Step 3

$$4x\overline{)24x^2-40x+16}\quad 6x-10____$$
$$-24x^2$$
$$0\ -40x$$
$$+40x$$
$$0\ +16$$

In Step 3, Professor K repeats the process described in Step 2, determining that $4x \bullet -10 = -40x$, placing a -10 in the quotient row, multiplying, subtracting ($-40x - (-40x) = -40x + 40x = 0$), and bringing down the 16.



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Step 4

$$
\begin{array}{r}
6x - 10 + 0 \\
4x\overline{)\,24x^2 - 40x + 16} \\
\underline{-24x^2} \\
0 - 40x \\
\underline{+40x} \\
0 + 16 \\
\underline{0} \\
16
\end{array}
$$

In Step 4, Professor K repeats the process described in Steps 2 and 3, determining that $4x$ cannot divide 16. Professor K places a 0 in the quotient row, then multiplies and subtracts ($4x \bullet 0 = 0$ and $16 - 0 = 16$). A remainder of 16 exists and Professor K places the remainder in the numerator of a fraction with the divisor ($4x$) as the denominator and reduces the fraction, $^{16}/_{4x} = {}^{4}/_{x}$. Professor K has determined through long division that the quotient of ($24x^2 - 40x + 16$) and ($4x$) is $6x - 10 + {}^{4}/_{x}$.

By following the steps in these three examples, you will be able to divide other polynomials of multiple terms by monomials. Show Professor K that you understand how to divide polynomials by monomials by working the activities for this lesson.

Life Principle



"We function in a manner consistent with our character."

—D. Frank McLane

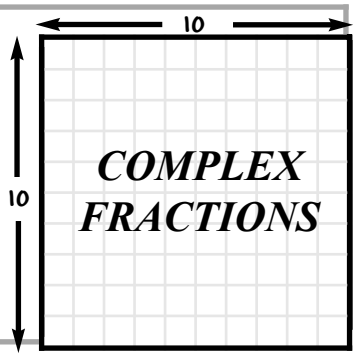
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Fractions

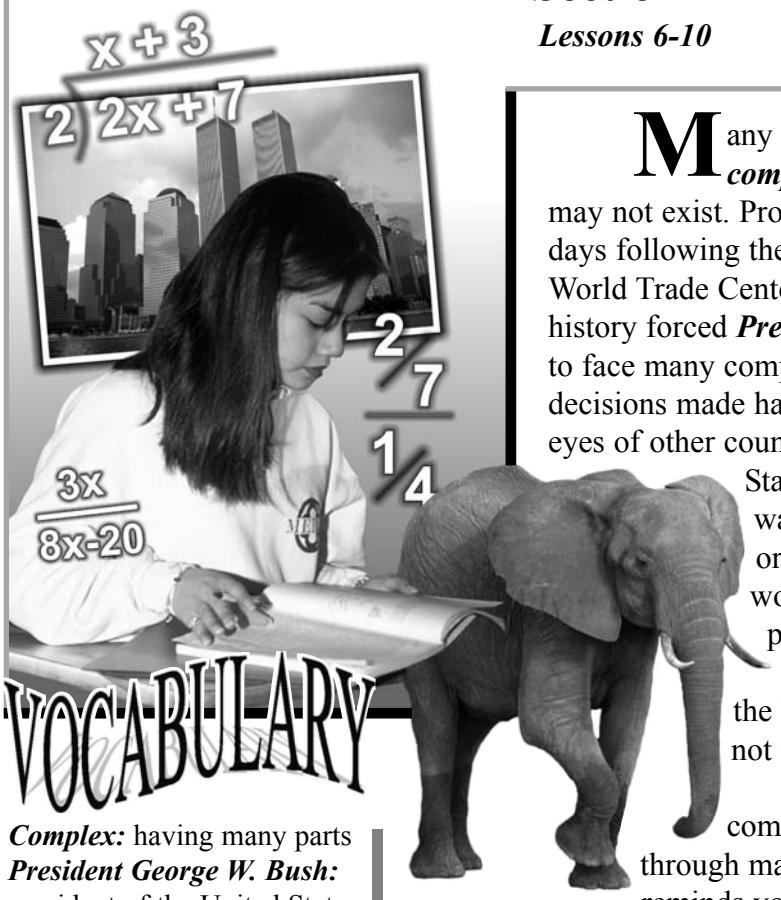
Lesson 7

Chapter 6



Section 2

Lessons 6-10





Professor K

Many problems in life are ***complex***. An easy solution may not exist. Professor K remembers the days following the terrorists' attacks on the World Trade Center. This time in America's history forced ***President George W. Bush*** to face many complex problems. The decisions made had to be well thought out. In the eyes of other countries of the world, if the United States ***retaliated*** in an unjustifiable way toward the terrorist organizations, then the United States would lose the support of other world powers. President G. W. Bush had to plan very carefully the course that the United States would take in order not to provoke other countries to anger.

You will face what appear to be complex problems in your "journey" through mathematics. However, Professor K reminds you that if you take the bigger, more complex problems one small step at a time you can work your way through the entire larger problem. This lesson will involve problems that contain complex fractions. Professor K will show you how to work through these problems one small step at a time.

In previous math courses, you were instructed about mathematical operations on fractions. For your convenience, in **Example Set 1**, Professor K will quickly review these operations with you.

VOCABULARY

Complex: having many parts
President George W. Bush: president of the United States during World Trade Center terrorist attacks
Retaliated: to repay in like kind
Reciprocal: the inverse or reversed position of a number; the direct opposite of a thing; having been turned upside down

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Example Set 1

a) $\frac{3}{7}+\frac{2}{7}=\frac{3+2}{7}=\frac{5}{7}$

a) Professor K adds fractions with the same (like) denominators, by placing the sum of the numerators over the denominator.

b) $\frac{4}{5}-\frac{2}{5}=\frac{4-2}{5}=\frac{2}{5}$

b) Professor K subtracts fractions with the same (like) denominators, by placing the difference of the numerators over the denominator.

c) $\frac{2}{5}\times\frac{6}{7}=\frac{2\times 6}{5\times 7}=\frac{12}{35}$

c) Professor K multiplies fractions by placing the product of the numerators over the product of the denominator. When multiplying, the denominators do not have to be the same as with addition and subtraction.

d) $\frac{6}{11}\div\frac{6}{7}=\frac{\cancel{6}}{11}\times\frac{7}{\cancel{6}}=\frac{7}{11}$

d) Professor K divides fractions by first inverting (flipping) the fraction to the right of the division operation symbol. Professor K then multiplies the first fraction and the inverted fraction by placing the product of the numerators over the product of the denominator. Professor K cancels or divides out the 6.

Some fractions are more complex than others. A complex fraction is a fraction that contains a fraction in either or both the numerator or denominator. Look at **Example Set 2**, as Professor K shows you how to simplify a problem containing complex fractions.



DEFINITION: COMPLEX FRACTION

A complex fraction is a fraction that contains a fraction in either or both the numerator or denominator.



Example Set 2

Simplify the following numerical expression.

$$\frac{3/4}{2}$$

The numerical expression shows the fraction three-fourths in the numerator and the number 2 in the denominator. Remember, that fractions are another way to show division.

Step 1 $\frac{3}{4}\div 2$

In Step 1, Professor K changes the fraction into a division problem, placing the denominator (as the divisor) to the right of the division symbol.

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Step 2 $\frac{3}{4}\times\frac{1}{2}$

In Step 2, Professor K inverts or flips the 2 ($^2/_1$), resulting in its ***reciprocal*** of $^1/_2$. This fraction is multiplied times $^3/_4$.

Step 3 $\frac{3\times1}{4\times2}=\frac{3}{8}$

In Step 3, Professor K performs the multiplication of the fractions by placing the product of the numerators in the numerator of the answer, and the product of the denominators in the denominator of the answer: $^3/_8$.

Example Set 3 shows how Professor K simplifies a complex fraction that contains a fraction in the denominator.

Example Set 3

Simplify the following complex fraction.

$\frac{5}{1/3}$

The numerical fraction shows the number 5 in the numerator and the fraction $^1/_3$ in the denominator.

Step 1 $5\div\frac{1}{3}$

In Step 1, Professor K changes the complex fraction into a division problem.

Step 2 5×3

In Step 2, Professor K inverts or flips the fraction ($^1/_3$), resulting in its reciprocal: $^3/_1$ or 3.

Step 3 $5 \times 3 = 15$

In Step 3, Professor K performs the multiplication to determine the simplification of the original complex fraction to be 15.

In **Example Set 4**, Professor K shows you how to simplify a complex fraction containing a fraction in both the numerator and denominator.

Example Set 4

Simplify the following complex fraction.

$\frac{2/7}{1/4}$

The complex numerical fraction shows the fraction, $^2/_7$ in the numerator and the fraction $^1/_4$ in the denominator.

Step 1 $\frac{2}{7}\div\frac{1}{4}$

In Step 1, Professor K changes the complex fraction into a division problem.

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Step 2 $\frac{2}{7}\times\frac{4}{1}$

In Step 2, Professor K inverts the fraction ($^1/_4$), resulting in its reciprocal: $^4/_1$.

Step 3 $\frac{2}{7}\times\frac{4}{1}=\frac{2\times4}{7\times1}=\frac{8}{7}$

In Step 3, Professor K multiplies to determine the simplification of the original complex fraction, then simplifies the answer: $^8/_7 = 1^1/_7$.

Show Professor K that you understand how to simplify complex fractions by working the activities for this lesson.

LIFE PRINCIPLE



"Being truthful when you know it will cost you is the true test of honesty."

—Anonymous

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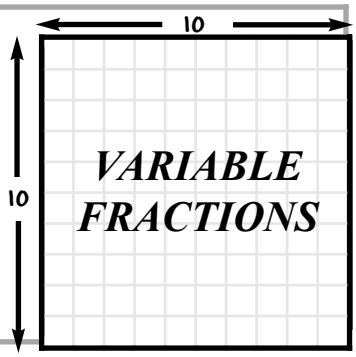
Lesson 8

Algebra

Chapter 6

Section 2

Lessons 6-10







Professor K

Professor K stated in the previous lesson, that many problems in life are complex. Often these problems are not only complex but also contain variables. The ***outcome*** of the situation may change, depending on what you do. *How you deal with a problem* is one of the variables in problem solving. Approaching a problem with the attitude that he can solve the problem, usually works for Professor K. Professor K also sees every problem in life as a challenge and an opportunity for success.

In this lesson, Professor K will show you how to simplify complex fractions containing variables. Look at the examples to see how Professor K simplifies such fractions and ***expresses*** the solution. In **Example Set 1**, Professor K shows you how to perform the basic mathematical operations on fractions containing variables.

Example Set 1

Perform the indicated operation.

a) $\frac{x}{7}+\frac{2}{7}=\frac{x+2}{7}$

a) Professor K adds fractions with the same (like) denominators, by placing the sum of the numerators over the denominator. You cannot add letters and numbers, so the numerator remains a mathematical expression, $x + 2$.

VOCABULARY

Outcome: results of a situation as it is solved

Expresses: states through spoken or written word

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b) $\frac{x}{5} - \frac{2}{5} = \frac{x-2}{5}$

b) Professor K subtracts fractions with the same (like) denominators, by placing the difference of the numerators over the denominator. As in part a), since you *cannot* combine letters and numbers when adding or subtracting, the numerator remains a mathematical expression, x - 2.

c) $\frac{x}{5} \bullet \frac{6}{7} = \frac{x \bullet 6}{5 \bullet 7} = \frac{6x}{35}$

c) Professor K multiplies fractions by placing the product of the numerators over the product of the denominator. When multiplying, the denominators do not have to be the same as with addition and subtraction. Remember, numbers and letters *can* be combined in multiplication to form one term.

d) $\frac{x}{11} \div \frac{6}{7} = \frac{x}{11} \bullet \frac{7}{6} = \frac{7x}{66}$

d) Professor K divides fractions by first inverting (flipping) the divisor (the fraction to the right of the division operation symbol). Professor K then multiplies the first fraction and the inverted fraction by placing the product of the numerators over the product of the denominators. Since the operation in the numerator is multiplication, Professor K combines the letter and number as shown.

Complex fractions that contain variables are simplified using the same rules as numerical complex fractions. Look at **Example Set 2** as Professor K shows you an example of how to simplify an algebraic expression containing complex fractions. Notice that the variable is in the denominator of this complex fraction.

Example Set 2

Simplify the following algebraic expression.

$$\frac{3/4}{x}$$

The algebraic expression shows the fraction three-fourths in the numerator and the variable (x) in the denominator. Remember that fractions are another way to show division.

Step 1 $\frac{3}{4} \div x$

In Step 1, Professor K changes the fraction into a division problem, placing the denominator to the right of the division symbol.

Step 2 $\frac{3}{4} \bullet \frac{1}{x}$

In Step 2, Professor K inverts the x or $x/1$. The resulting reciprocal is $1/x$. This fraction is multiplied times $3/4$.

Step 3 $\frac{3 \bullet 1}{4 \bullet x} = \frac{3}{4x}$

In Step 3, Professor K performs the multiplication of the fractions by placing the product of the numerators in the numerator of the answer, and the product of the denominators in the denominator of the answer: $3/4x$.

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Example Set 3 shows how Professor K simplifies a complex fraction that contains a variable and a fraction in the denominator.

Example Set 3

Simplify the following complex fraction.

$$\frac{x}{1/3}$$

The algebraic expression shows the variable, x, in the numerator and the fraction, $1/3$, in the denominator.

Step 1 $x \div \frac{1}{3}$

In Step 1, Professor K changes the complex fraction into a division problem.

Step 2 $x \bullet 3$

In Step 2, Professor K inverts the fraction, $1/3$, resulting in its reciprocal of $3/1$ or 3.

Step 3 $x \bullet 3 = 3x$

In Step 3, Professor K performs the multiplication to determine the simplification of the original complex fraction to be $3x$.

In **Example Set 4**, Professor K shows you how to simplify a complex fraction containing a fraction in both the numerator and denominator. One of the fractions contains a variable.



Example Set 4

Simplify the following complex fraction.

$$\frac{2/3}{x/4}$$

The complex fraction shows the fraction, $2/3$, in the numerator and the fraction, $x/4$, in the denominator.

Step 1 $\frac{2}{3} \div \frac{x}{4}$

In Step 1, Professor K changes the complex fraction into a division problem.

Step 2 $\frac{2}{3} \bullet \frac{4}{x}$

In Step 2, Professor K inverts the fraction, $x/4$, resulting in its reciprocal: $4/x$.

Step 3 $\frac{2}{3} \bullet \frac{4}{x} = \frac{2 \bullet 4}{3 \bullet x} = \frac{8}{3x}$

In Step 3, Professor K performs the multiplication to determine the simplification of the original complex fraction to be $8/3x$.

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Show Professor K that you understand how to simplify complex fractions containing variables by working the activities for this lesson.

Life Principle



"The memories of tomorrow depend on your attitude today."

—Dave Weinbaum

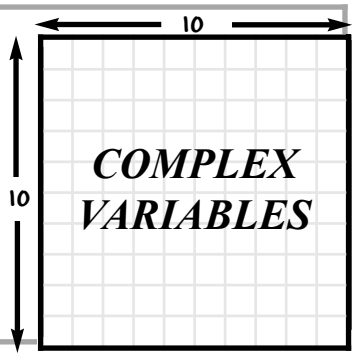
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Lesson 9

Algebra

Chapter 6



Section 2

Lessons 6-10



In Lessons 2 and 3 of this chapter, Professor K presented a review of the Distributive Property. Professor K uses the Distributive Property to simplify the complex fractions in this lesson. Remember, mathematics is similar to life. You must follow certain rules and you must continually remember and draw from previous knowledge and experiences to determine solutions to new problems. Look at the examples to see how Professor K simplifies such fractions and expresses the solutions. In **Example Set 1**, Professor K shows you how to perform the basic mathematical operations on fractions containing grouping symbols.

Example Set 1

Perform the indicated operations.

$$\frac{2(x+1)}{7}+\frac{3(x-2)}{7}$$

This problem consists of the addition of two fractions, containing algebraic expressions in the numerators.

VOCABULARY

Reciprocal: the inverse of a number; a number is turned upside down: as in the reciprocal of 2 is $^1/_2$ and the reciprocal of $^1/_3$ is 3.

Step 1 $\dfrac{2\bullet x+2\bullet 1}{7}+\dfrac{3\bullet x-3\bullet 2}{7}$

Professor K uses the Distributive Property to simplify the numerators.

Step 2 $\dfrac{2x+2}{7}+\dfrac{3x-6}{7}$

Professor K performs the multiplications in Step 1.

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Step 3	$\frac{2x+2+3x-6}{7}$	Professor K adds fractions with the same (like) denominators in Step 2, by placing the sum of the numerators over the denominator.
Step 4	$\frac{2x+3x+2-6}{7}$	Professor K rearranges the numerator using the Commutative Property.
Step 5	$\frac{(2+3)x+(2-6)}{7}$	Professor K groups the like terms for simplification.
Step 6	$\frac{5x-4}{7}$	Professor K performs the operations in Step 5 and determines the final simplification of the original algebraic expression. Notice that 2 - 6 = -4. Adding a negative number means to *subtract* that number, $5x + (-4) = 5x - 4$.

Look at **Example Set 2** as Professor K shows you an example of how to simplify an algebraic expression containing complex fractions. Notice that Professor K must use the Distributive Property before continuing the simplification.

Example Set 2

Simplify the following algebraic expression.

$$\frac{2(x+3)/4}{x}$$



Step 1	$\frac{2 \bullet x + 2 \bullet 3}{4} = \frac{2x+6}{4}$	In Step 1, Professor K simplifies the numerator using the Distributive Property as shown.
Step 2	$\frac{2x+6}{4} \div x$	In Step 2, Professor K changes the original complex fraction into a division problem, placing the denominator to the right of the division symbol.
Step 3	$\frac{2x+6}{4} \bullet \frac{1}{x}$	In Step 3, Professor K inverts the x or $x/1$. The resulting ***reciprocal*** is $1/x$. This fraction is multiplied times the first fractional term.
Step 4	$\frac{(2x+6) \bullet 1}{4 \bullet x} = \frac{2x+6}{4x}$	In Step 4, Professor K performs the multiplication of the fractions by placing the product of the numerators in the numerator of the answer, and the product of the denominators in the denominator of the answer.

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Example Set 3 shows how Professor K simplifies another complex fraction that contains a fraction in the denominator requiring the use of the Distributive Property.

Example Set 3

Simplify the following complex fraction.

$$\frac{x}{4(2x-5)/3}$$

The algebraic expression shows the variable (x) in the numerator and a fractional expression in the denominator.

Step 1 $x \div \frac{4(2x-5)}{3}$

In Step 1, Professor K changes the complex fraction into a division problem.

Step 2 $\frac{x}{1} \bullet \frac{3}{4(2x-5)}$

In Step 2, Professor K inverts the fraction, $^{4(2x-5)}/_{3}$. The resulting reciprocal is $^{3}/_{4(2x-5)}$. Professor K also changes x to $^{x}/_{1}$.

Step 3 $\frac{x \bullet 3}{1 \bullet 4(2x-5)} = \frac{3x}{4(2x-5)}$

In Step 3, Professor K performs the multiplications.

Step 4 $\frac{3x}{4(2x-5)} = \frac{3x}{4 \bullet 2x - 4 \bullet 5} = \frac{3x}{8x-20}$

Professor K uses the Distributive Property to determine the simplification of the original complex fraction to be $^{3x}/_{8x - 20}$.

Show Professor K that you understand how to simplify complex fractions containing variables by working the activities for this lesson.

LIFE PRINCIPLE



"All time spent being angry
is time lost being happy."
—Mexican Proverb

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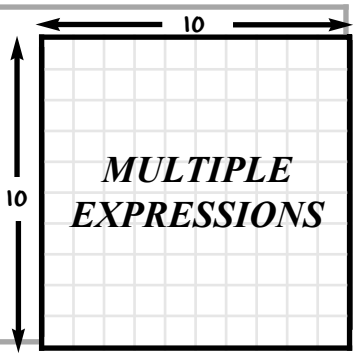
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Lesson 10

Algebra

Chapter 6



Section 2

Lessons 6-10





At first glance, many problems in life seem too large to solve. However, remember when Professor K sees the ***magnitude*** of a problem, he approaches the problem a little at a time. Professor K once heard the story about the man who went on ***safari*** to Africa. The man came across a native who had killed an elephant. The man asked the native what he was going to do with the elephant. The native said that he would use the elephant for food. The man then asked the native how he thought that he could eat such a big animal. The native replied, "One bite at a time."

This story shows how you should approach mathematic problems that seem difficult and complex. You should work toward the final answer one "bite" (step) at a time. In this lesson, Professor K will take you through problems containing complex fractions and algebraic terms and expressions. Professor K will show you how to apply the steps you have learned about simplifying complex fractions to solving other complex expressions, *step by step*. Look at **Example Set 1** as Professor K simplifies a complex fraction.



Magnitude: greatness of size or amount
Safari: a journey or trip; usually in Africa for hunting or picture taking

Example Set 1

Simplify.

$$\frac{\dfrac{4(3x-2)}{6(x+5)}}{\dfrac{2(2x+3)}{5(4x-2)}}$$

This complex fraction is not difficult to simplify if you take it apart and work with it one small "bite" at a time. Professor K will use the procedures described in the previous lessons to accomplish this.

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Step 1

$$\frac{4(3x-2)}{6(x+5)}=\frac{4\bullet 3x-4\bullet 2}{6\bullet x+6\bullet 5}=$$

$$\frac{12x-8}{6x+30}$$

In Step 1, Professor K uses the Distributive Property to simplify the numerator of the complex fraction.

Step 2

$$\frac{2(2x+3)}{5(4x-2)}=\frac{2\bullet 2x+2\bullet 3}{5\bullet 4x-5\bullet 2}=$$

$$\frac{4x+6}{20x-10}$$

In Step 2, Professor K uses the Distributive Property to simplify the denominator of the complex fraction.

Step 3

$$\frac{12x-8}{6x+30}\div\frac{4x+6}{20x-10}$$

In Step 3, Professor K places the solutions obtained in Steps 2 and 3 in the form of a division problem.

Step 4

$$\frac{12x-8}{6x+30}\bullet\frac{20x-10}{4x+6}=$$

$$\frac{(12x-8)\bullet(20x-10)}{(6x+30)\bullet(4x+6)}$$

In Step 4, Professor K follows the procedure for division of fractions by making the division problem a multiplication problem; he inverts the fraction to the right of the division symbol.

Step 5

$$\frac{(12x-8)\bullet(20x-10)}{(6x+30)\bullet(4x+6)}=$$

$$\frac{240x^2-280x+80}{24x^2+156x+180}$$

In Step 5, Professor K uses the Distributive Property in the numerator and FOIL in the denominator to determine the final answer.

Look at **Example Set 2** as Professor K shows you an example of how to simplify an algebraic expression containing complex fractions. Notice that Professor K must use the Distributive Property before continuing the simplification.

Example Set 2

Simplify.

$$\frac{\dfrac{2(3x+1)}{3(x+2)}}{\dfrac{3(x+3)}{2(x-2)}}$$

This complex fraction is not difficult to simplify if you take it apart and work with it one small "bite" at a time. Professor K will use the procedures described in the previous lessons to accomplish this.

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Step 1 $\frac{2(3x+1)}{3(x+2)} = \frac{2 \bullet 3x + 2 \bullet 1}{3 \bullet x + 3 \bullet 2} = \frac{6x+2}{3x+6}$

In Step 1, Professor K uses the Distributive Property to simplify the numerator of the complex fraction.

Step 2 $\frac{3(x+3)}{2(x-2)} = \frac{3 \bullet x + 3 \bullet 3}{2 \bullet x - 2 \bullet 2} = \frac{3x+9}{2x-4}$

In Step 2, Professor K uses the Distributive Property to simplify the denominator of the complex fraction.

Step 3 $\frac{6x+2}{3x+6} \div \frac{3x+9}{2x-4}$

In Step 3, Professor K places the solutions obtained in Steps 2 and 3 in the form of a division problem.

Step 4 $\frac{6x+2}{3x+6} \bullet \frac{2x-4}{3x+9} = \frac{(6x+2)(2x-4)}{(3x+6)(3x+9)}$

In Step 4, Professor K follows the procedure for division of fractions by making the division problem a multiplication problem by inverting the fraction to the right of the division symbol.

Step 5 $\frac{(6x+2)(2x-4)}{(3x+6)(3x+9)} = \frac{12x^2 - 20x - 8}{9x^2 + 45x + 54}$

In Step 5, Professor K uses the Distributive Property in the numerator and FOIL in the denominator to determine the final answer.

Show Professor K that you understand how to simplify complex fractions containing variables by working the activities for this lesson.

Life Principle



"Life is a journey. It is best to plan where you are going."

—Anonymous

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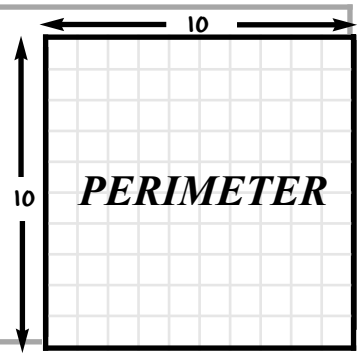


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Lesson 11

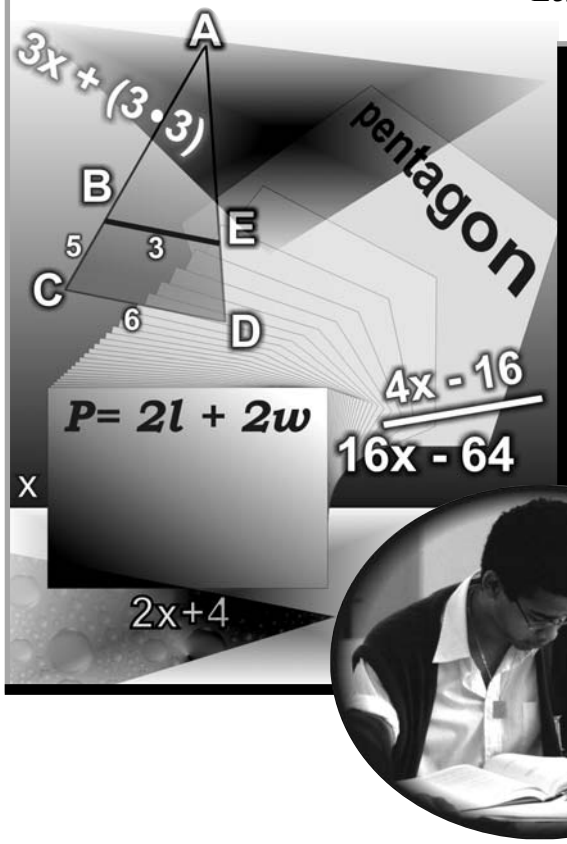
Geometry

Chapter 6



Section 3

Lessons 11-15



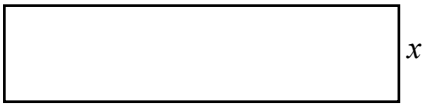
The lengths of the sides of shapes are sometimes given in the form of polynomials. You can use what you have learned about calculating the ***perimeter*** of shapes and how to add polynomials to calculate dimensions. Look at **Example Set 1** as Professor K shows you how to determine the perimeter of shapes whose lengths are shown as polynomials.



Professor K

Example Set 1

The length of a rectangle is four more than twice the width. If the perimeter of the rectangle is 20 cm, calculate the dimensions of the rectangle.



Step 1

Let x = width and $2x + 4$ = length of the rectangle

In Step 1, Professor K transforms the description of the dimensions of the rectangle into mathematical terms.

Step 2

$P = 2l + 2w$

$20 = 2(2x + 4) + 2x$

In Step 2, Professor K writes the formula for the perimeter of a rectangle and then substitutes the values from Step 1 for l, w and P.

VOCABULARY

Perimeter: the distance around a shape
Rhs: right hand side
Integer: member of the set of numbers {…-3, -2, -1, 0, 1, 2, 3, …}

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Step 3	$20 = 4x + 8 + 2x$	In Step 3, Professor K uses the Distributive Property to begin to solve the problem.
Step 4	$20 = 4x + 2x + 8$	In Step 4, Professor K rearranges the ***rhs*** of the equation $20 = 6x + 8$ by using the Commutative Property of Addition and combines like terms.
Step 5	$20 - 8 = 6x + 8 - 8$ $12 = 6x$	In Step 5, Professor K subtracts eight from both sides of the equation to begin isolation of the x.
Step 6	$^{12}/_{6} = {}^{6x}/_{6} \rightarrow x = 2$	In Step 6, Professor K divides both sides of the equation by six to complete the isolation of x.
Step 7	width = x = 2 cm length = $2x + 4$ = $2 \bullet 2 + 4 = 8$ cm	In Step 7, Professor K substitutes the value of x calculated in Step 6 into the polynomials for the width and length from Step 1 to determine that the width of the rectangle is 2 cm and the length is 8 cm.

Example Set 2 shows how Professor K calculates the perimeter of a triangle in which the dimensions are given as polynomials.

Example Set 2

The perimeter of a triangle is 63 inches. Calculate the dimensions of the triangle if the sides are consecutive ***integers***.

Step 1	Let x = first side (s_1) $x + 1$ = second side (s_2) $x + 2$ = third side (s_3) of the triangle.	In Step 1, Professor K transforms the description of the dimensions of the triangle into mathematical terms.
Step 2	$P = s_1 + s_2 + s_3$ $63 = x + x + 1 + x + 2$	In Step 2, Professor K writes the formula for the perimeter of a triangle and then substitutes the values from Step 1 for s_1, s_2 and s_3.
Step 3	$63 = 3x + 3$	In Step 3, Professor K combines *like terms*.

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Step 4	$63 - 3 = 3x + 3 - 3 \rightarrow$ $60 = 3x$	In Step 4, Professor K subtracts three from both sides of the equation to begin isolation of the x.
Step 5	$^{60}/_{3} = {}^{3x}/_{3} \rightarrow x = 20$	In Step 5, Professor K divides both sides of the equation by three to complete the isolation of x.
Step 6	side one = x = 20 inches side two = $x + 1$ = 20 + 1 = 21 inches side three = $x + 2$ = 20 + 2 = 22 inches	In Step 6, Professor K substitutes the value of x calculated in Step 6 into the polynomials for the sides of the triangle from Step 1 to determine that side one = 20 inches, side two = 21 inches and side three = 22 inches.

You can calculate the perimeter of various shapes by following the same steps as in these examples. Show Professor K that you understand how to calculate the perimeter of shapes by working the activities for this lesson.

LIFE PRINCIPLE



"It is better to ask some of the questions than to know all of the answers."

—James Thurber

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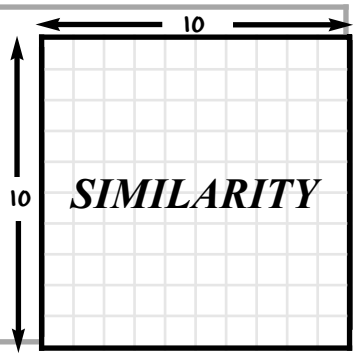


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Lesson 12

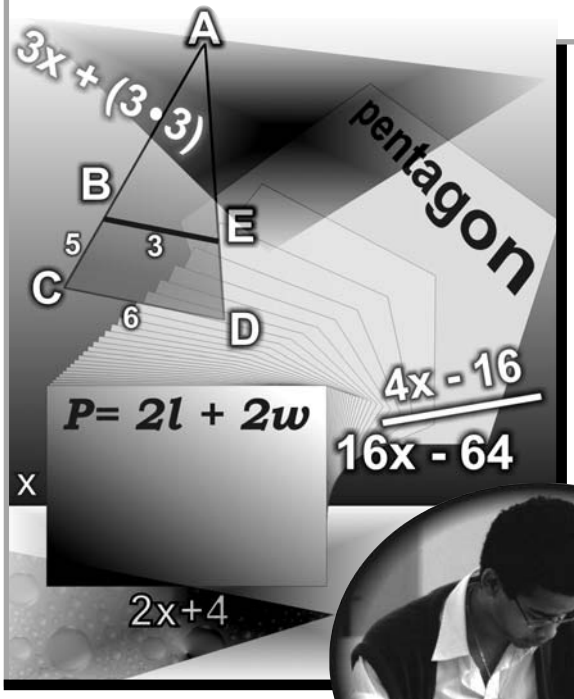
Geometry

Chapter 6



Section 3

Lessons 11-15



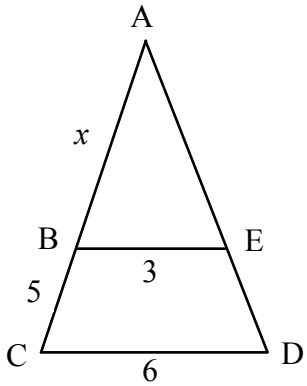


In the previous lesson you learned that the lengths of the sides of shapes are sometimes given in the form of polynomials. You can use what you have learned about calculating the lengths of the sides of shapes in similarity problems to multiply and divide polynomials to make calculations. Look at **Example Set 1** as Professor K shows you how to make calculations when the lengths of the sides of similar shapes are polynomials.

Example Set 1

The length of one leg of a triangle is five units more than the length of the ***corresponding*** leg of another similar ***isosceles*** triangle. The base of the smaller triangle is 3 cm in length and the base of the larger triangle is 6 cm. Calculate the lengths of the legs of the triangles.

Step 1 Professor K draws a picture of the two triangles.



VOCABULARY

Corresponding: in the same place of another figure
Isosceles: two sides equal

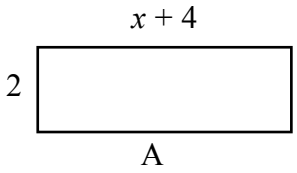
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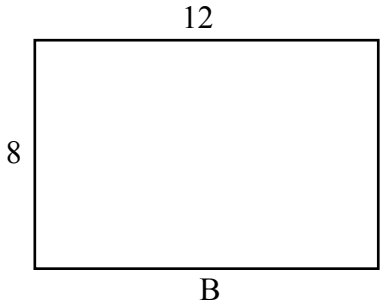
Step 2	$\frac{AB}{AC} = \frac{BE}{CD} \rightarrow \frac{x}{x+5} = \frac{1}{2} \rightarrow$ $2x = 1 \bullet (x + 5) \rightarrow 2x = x + 5$	Professor K sets up a proportion between the sides of the two triangles and cross-multiplies.
Step 3	$2x = x + 5 \rightarrow$ $2x - x = x + 5 - x \rightarrow$ $x = 5$	Professor K isolates the x on the lhs of the equation by subtracting x from both sides and determines that $x = 5$.
Step 4	AB = x = 5 cm AC = x + 5 = 10 cm	Professor K substitutes the value of x into the expressions for the length of the sides.

Example Set 2 shows how Professor K calculates the length of the side of a rectangle using similarity principles. The dimensions are given as polynomials.

Example Set 2

Calculate the length of the sides of rectangle A. Rectangle A is similar to rectangle B.





Step 1	$^{2}/_{(x+4)} = {}^{8}/_{12}$	In Step 1, Professor K sets up a proportion between the sides of rectangle A and rectangle B.
Step 2	$12(2) = 8(x + 4) \rightarrow$ $24 = 8x + 32$	In Step 2, Professor K cross-multiplies and uses the Distributive Property on the rhs of the equation.
Step 3	$24 - 32 = 8x + 32 - 32$ $-8 = 8x$	In Step 3, Professor K subtracts 32 from both sides of the equation.
Step 4	$^{-8}/_{8} = {}^{8x}/_{8} \rightarrow x = -1$	In Step 4, Professor K divides both sides by eight to determine that the value of x is -1.

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Step 5	$x + 4 = -1 + 4 = 3$ inches	In Step 5, Professor K substitutes the value of x calculated in Step 4 into the polynomial for the side of the rectangle to determine that the dimensions of rectangle A is 2 inches by 3 inches.

You can calculate the dimensions of various shapes by following the same steps as in these examples. Show Professor K that you understand how to calculate the dimensions of various of shapes by working the activities for this lesson.

LIFE PRINCIPLE



"If we do not maintain Justice,
Justice will not maintain us."

—Francis Bacon (1909-1992)

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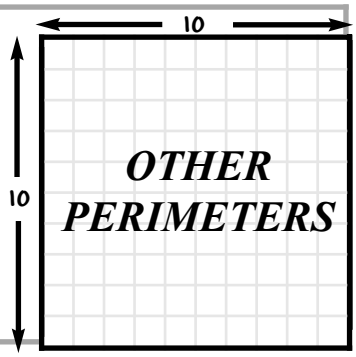
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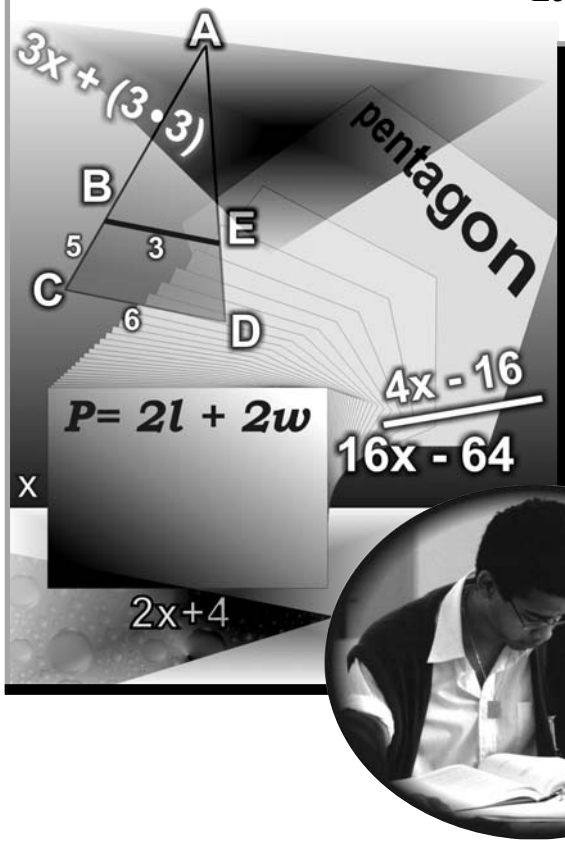
Lesson 13

Chapter 6

Section 3

Lessons 11-15



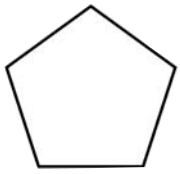


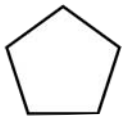


You learned in Lesson 11 that the lengths of the sides of shapes are sometimes given in the form of polynomials. You can use what you have learned about calculating the perimeter of shapes and how to add polynomials to make calculations of shapes other than triangles and rectangles. Look at **Example Set 1** as Professor K shows you how to determine the perimeter of shapes whose lengths are shown as polynomials.

Example Set 1

The length of the side of an ***equilateral pentagon*** is five inches more than the length of the side of a second equilateral pentagon. If the perimeter of the second pentagon is 20 inches, calculate the perimeter of the first pentagon.





VOCABULARY

Equilateral: all the sides of equal length
Pentagon: figure with 5 sides

Step 1

Let x = length of the side of a second pentagon; $x + 5$ = length of the side of the first pentagon

In Step 1, Professor K transforms the description of the dimensions of the pentagons into mathematical terms.

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Step 2	$P = 5s$ $20 = 5x$	In Step 2, Professor K writes the formula for the perimeter of a pentagon and then substitutes the values from Step 1 for s for the second pentagon.
Step 3	$^{20}/_{5} = {}^{5x}/_{5} \rightarrow 4 = x$	In Step 3, Professor K solves for x by dividing both sides by 5 to determine that the length of the side of the second pentagon is 4 inches.
Step 4	$x + 5 = 4 + 5 = 9$ inches	In Step 4, Professor K places the answer from Step 3 into the expression for the length of the side of the first pentagon
Step 5	$P = 5s = 5(9) = 45$ inches	In Step 5, Professor K places the answer from Step 4 into the formula for the perimeter of a pentagon to determine that the perimeter of the first pentagon is 45 inches.

Example Set 2 shows how Professor K calculates the circumference of a circle in which the radius is given as a polynomial.

Example Set 2

The circumference of a circle is 44 feet. The radius of the circle is given in the form of the polynomial x - 2. Calculate the diameter of the circle.

Step 1	$C = 2\pi r$	In Step 1, Professor K writes the formula for the circumference of a circle. Remember, pi (π) equals 3.14 or $^{22}/_{7}$.
Step 2	$44 = 2(^{22}/_{7})(x - 2)$	In Step 2, Professor K substitutes the numbers given in the problem into the proper locations. Professor K uses $^{22}/_{7}$ as the value of π.
Step 3	$2(^{22}/_{7})(x - 2) = {}^{2}/_{1}(^{22}/_{7})^{(x - 2)}/_{1}$ $\frac{2(22)(x-2)}{1 \bullet 7 \bullet 1} = \frac{44(x-2)}{7} =$ $\frac{44x-88}{7}$	In Step 3, Professor K changes the numbers on the rhs of the equation to fractions and multiplies the resulting fractions together using the rules for multiplying fractions and the Distributive Property.



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Step 4 $44 = \frac{44x - 88}{7}$

In Step 4, Professor K places the answer for the rhs from Step 3 into the equation.

Step 5 $7 \bullet 44 = \frac{44x - 88}{\cancel{7}} \bullet \cancel{7}$

In Step 5, Professor K multiplies both sides of the equation by seven to begin isolation of x.

Step 6

$308 = 44x - 88$
$308 + 88 = 44x + 88$
$396 = 44x$
$^{396}/_{44} = {}^{44x}/_{44}$
$9 = x$

In Step 6, Professor K performs the calculations from Step 5, adds eighty-eight to both sides and divides by 44 to determine that $x = 9$. Placing this calculation into the polynomial for the radius of the circle, $x - 2 = 9 - 2$ equals 7 inches. The diameter, d, of the circle is $d = 2r = 2 \bullet 7$ equals 14 inches.

You can calculate the perimeter of various shapes by following the same steps as in these examples. Show Professor K that you understand how to calculate the perimeter of shapes by working the activities for this lesson.

LIFE PRINCIPLE



"Courage is resistance to fear,
mastery of fear—not absence of fear."
—Mark Twain (1835-1910)

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x+3=7

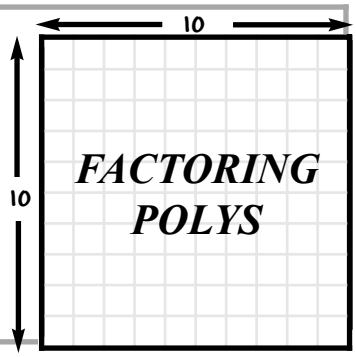
Lesson 14

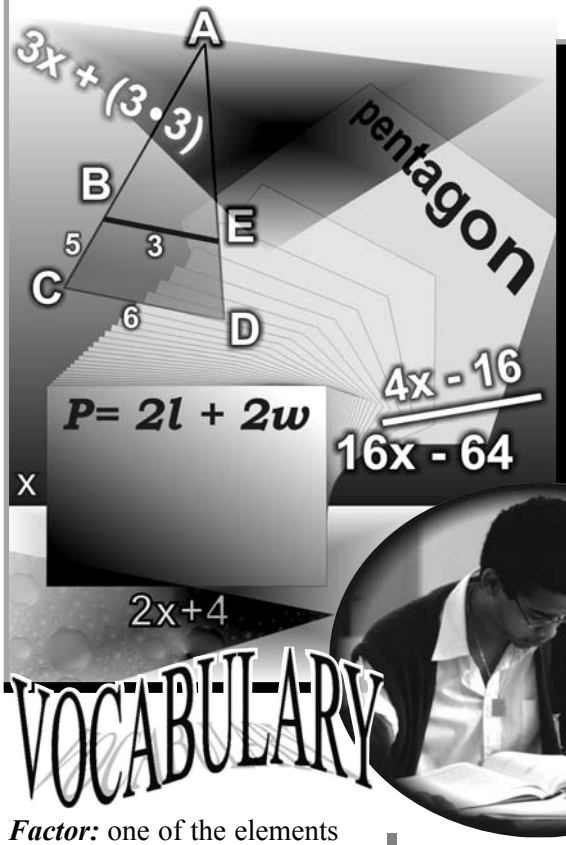
Algebra

Chapter 6

Section 3

Lessons 11-15





VOCABULARY

Factor: one of the elements contributing to a particular result or situation; one of the numbers multiplied together to obtain a product; to determine the prime factors of a mathematic term or expression; the opposite operation of the Distributive Property

Complete factorization: determining all the prime factors of a number or mathematical expression



Problems in life usually are not simple with only one ***factor*** as the cause. These problems are similar to problems in mathematics. You must begin to break the problem up into all the separate factors to reduce or solve the problem. Remember that *prime numbers* are numbers that contain only two factors: one and the number itself ($2 = 1 \bullet 2$; $3 = 1 \bullet 3$; $5 = 1 \bullet 5$). Composite numbers are numbers that are *not* prime, or contain factors other than one and the number itself ($4 = 2 \bullet 2$; $6 = 2 \bullet 3$). The ***complete factorization*** of 8 is *all of the prime factors multiplied together* or $8 = 2 \bullet 2 \bullet 2$.

The complete factorization of polynomials is similar to that of numbers. If possible, break the polynomial into all the prime factors. If the polynomial cannot be factored, then the polynomial is prime. What you will learn in this lesson is what Professor K likes to call the "un-distributing" procedure. Mathematicians call it *factoring*. Look at **Example Set 1** as Professor K refreshes your memory on determining the factors of a term.

Example Set 1

Factor the following terms.

a) 30

$3 \bullet 10$

$3 \bullet 5 \bullet 2$

a) Professor K divides 30 by 3 to obtain 10. He then divides 10 by 5 to obtain 2. Therefore, the prime factors of 30 are 3, 5, and 2.

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b) $24x^3y^2$
$(2 \bullet 12)x^3y^2$
$(2 \bullet 2 \bullet 6)x^3y^2$
$(2 \bullet 2 \bullet 2 \bullet 3)x^3y^2$
$(2 \bullet 2 \bullet 2 \bullet 3)(x \bullet x \bullet x)(y \bullet y)$

b) Professor K divides 24 by 2 to obtain 12. He then divides 12 by 2 to obtain 6. Professor K then divides 6 by 2 to obtain 3. Therefore, the prime factors of 24 are 2, 2, 2, 3. Professor K remembers that to raise to a power as $x^3\ y^2$ means to multiply the base times itself the number of times as the value or the exponent. Therefore, $x^3 = x \bullet x \bullet x$ and $y^2 = y \bullet y$.

Example Set 2 shows how Professor K uses the principles in **Example Set 1** to factor a binomial.

Example Set 2

Factor the following binomial.

$3x + 9$

Step 1	$3x + (3 \bullet 3)$	Professor K breaks the number nine up into its prime factors, $3 \bullet 3$.
Step 2	$3(x + 3)$	Professor K notices that both terms in the binomial have a common factor, 3. Professor K "un-distributes," or more properly, *factors out* a 3 by dividing both terms of the binomial by 3 and places the divisor (3) on the outside of a set of parentheses placed around the remaining binomial.
Step 3	$3(x + 3) = 3 \bullet x + 3 \bullet 3 = 3x + 9$	Professor K checks his answer by using the Distributing Property and obtains the original binomial.

Notice that in checking the answer to the factorization problem in **Example Set 2**, Professor K used the Distributive Property. Just as in a division problem, check the answer by multiplying. This is why Professor K refers to factoring as "un-distributing." Look at **Example Set 3** as Professor K factors a trinomial.

Example Set 3

Factor the following polynomial.

$10xa + 24xb + 16xc$

Step 1	$(2 \bullet 5 \bullet x \bullet a) +$ $(3 \bullet 2 \bullet 2 \bullet 2 \bullet x \bullet b) +$ $(2 \bullet 2 \bullet 2 \bullet 2 \bullet x \bullet c)$	In Step 1, Professor K factors the numerical coefficients (numbers in front of the variables) of the three terms: $10 = 2 \bullet 5$; $24 = 3 \bullet 2 \bullet 2 \bullet 2$; $16 = 2 \bullet 2 \bullet 2 \bullet 2$. Also the variables in each term are shown in factored form.

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Step 2	$2x(5a + 12b + 8c)$	In Step 2, Professor K notices that $2 \bullet x$ is a factor common to all three terms. Professor K factors out $2x$ by dividing all of the terms of the trinomial by $2x$ and places the divisor, $2x$, on the outside of a set of parentheses placed around the remaining trinomial.
Step 3	$(2x \bullet 5a) + (2x \bullet 12b) + (2x \bullet 8c) = 10xa + 24xb + 16xc$	In Step 3, Professor K checks his answer using the Distributive Property and determines that the product, $10xa + 24xb + 16xc$ is the same as the original trinomial. Therefore, the factorization is correct.

You can factor various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to factor polynomials by working the activities for this lesson.

LIFE PRINCIPLE



"Nothing is swifter than rumor."

—Horace

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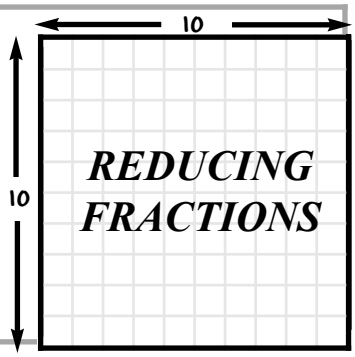
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Lesson 15

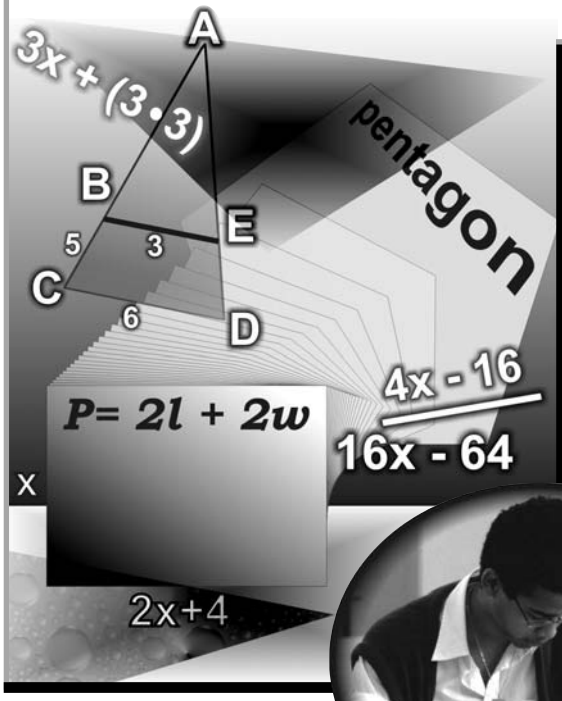
Algebra

Chapter 6



Section 3

Lessons 11-15



R**educing* problems to a more simplified form can make the solution to the problem easier to obtain. When you begin to ***factor out some of the things in your life that cause you problems, your life can become less stressful and more simplified. In the last lesson you learned how to factor some polynomials. Those procedures can assist you in reducing fractions that contain polynomials. Earlier in this chapter (Lesson 10, for example) you performed the operations and displayed your answers. Answers similar to those will be the problems for this lesson, and the problems for those lessons will be the answer to the problems in this lesson. Remember, earlier we were using the Distributive Property on polynomials, now we are factoring, which is the opposite of the Distributive Property. That is why the answers are the *problems* and the problems are the *answers*. To follow Professor K's thinking, look at **Example Set 1** as he reduces a fraction containing polynomials by first factoring the polynomials.



VOCABULARY

Reducing: making smaller or putting in a more simplified form

Factor: one of the elements contributing to a particular result or situation; one of the numbers multiplied together to obtain a product; to determine all the prime factors of a mathematic term or expression; the opposite operation of the Distributive Property

Example Set 1

Simplify the following.

$$\frac{8x+16}{16x+16}$$

Professor K will begin to simplify this expression by factoring the numerator and denominator separately.

Step 1 $8x + 16 = (8 \bullet x) + (8 \bullet 2)$
$8(x + 2)$

In Step 1, Professor K factors the numerator by determining the common factor to be 8.

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Step 2

$16x + 16$
$(16 \bullet x) + (16 \bullet 1)$
$16(x + 1)$

In Step 2, Professor K factors the denominator by determining the common factor to be 16.

Step 3

$$\frac{8(x+2)}{16(x+1)}$$
$$\frac{1(x+2)}{2(x+1)} = \frac{x+2}{2x+2}$$

In Step 3, Professor K places the factored expressions back in the numerator and denominator. He then reduced $^{8}/_{16} = {}^{1}/_{2}$. No other terms can be divided out or *canceled*; so, the simplified form of the original fraction after distributing the numbers in front of the binomials is $\frac{x+2}{2x+2}$.



Professor K reminds you that you cannot divide out a factor unless the factor is contained in all of the terms of the expression. In the fractional expression, from **Example Set 1**, the 2 in the numerator cannot reduce to one and the 2s in the denominator to ones, because there is no two as a coefficient of the x in the numerator. Professor K calls this *canceling* across an addition or subtraction symbol in a fraction. You cannot do this unless every term contains the same factor.

Example Set 2 shows how Professor K uses the principles in **Example Set 1** to factor a binomial.

Example Set 2

Simplify the following.

$$\frac{240x^2 - 160x}{24x^2 + 156x + 180}$$

Professor K will begin to simplify this expression by factoring the numerator and denominator separately.

Step 1

$240x^2 - 160x =$
$(80x \bullet 3x) - (80x \bullet 2)$
$80x(3x - 2)$

In Step 1, Professor K factors the numerator, determining the common factor to be $80x$.

Step 2

$24x^2 + 156x + 180 =$
$(12 \bullet 2x^2) + (12 \bullet 13x) + (12 \bullet 15)$
$12(2x^2 + 13x + 15)$

In Step 2, Professor K factors the denominator, determining the common factor to be 12.

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Step 3	$\frac{80x(3x-2)}{12(2x^2+13x+15)}$ $\frac{20x(3x-2)}{3(2x^2+13x+15)}$	In Step 3, Professor K places the factored expressions back in the numerator and denominator. He then reduced $^{80}/_{12}$ to $^{20}/_{3}$. No other terms can be divided out or canceled, so, the simplified form of the original fraction is $\frac{20x(3x-2)}{3(2x^2+13x+15)}$.

Example Set 3 shows how Professor K uses the principles in **Example Sets 1 and 2** to factor another fractional expression.

Example Set 3

Simplify the following.

$\frac{4x-16}{16x-64}$

Professor K will begin to simplify this expression by factoring the numerator and denominator separately.

Step 1	$4x$ - 16 = $(4 \bullet x)$ - $(4 \bullet 4)$ $4(x - 4)$	In Step 1 Professor K factors the numerator, determining the common factor to be 4.
Step 2	$16x$ - 64 = $(16 \bullet x)$ - $(16 \bullet 4)$ $16(x - 4)$	In Step 2, Professor K factors the denominator, determining the common factor to be 16.
Step 3	$\frac{4(x-4)}{16(x-4)} = \frac{4 \bullet 1(x-4)}{4 \bullet 4(x-4)} =$ $1 \bullet \frac{1}{4} \bullet 1 = \frac{1}{4}$	In Step 3, Professor K places the factored expressions back in the numerator and denominator. He then reduced $^{4}/_{4}$ to 1 and $^{(x-4)}/_{(x-4)}$ to 1. No other terms can be divided out or canceled, so, the simplified form of the original fraction is $^{1}/_{4}$.

Notice in Step 3 of **Example Set 3**, that when in parentheses, the (x - 4) is considered one term. Therefore, when the exact same mathematical expression is in the numerator and denominator, $^{(x-4)}/_{(x-4)}$, these expressions can divide out to become 1, just as 4 did in $^{4}/_{4}$.

You can factor various other polynomials by following the same steps as in these examples. Show Professor K that you understand how to factor polynomials by working the activities for this lesson.

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Life Principle



"I long to accomplish a great and noble task, but it is my chief duty to accomplish small tasks as if they were great and noble."

—Helen Keller (1880-1968)
Blind Educator and Writer

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Intermediate Math Skills

Mathematics Charts

LENGTH - ENGLISH SYSTEM

12 inches	=	1 foot
3 feet	=	1 yard
5.5 yards	=	1 rod
320 rods	=	1 mile
1,760 yards	=	1 mile
5,280 feet	=	1 mile

VOLUME - ENGLISH SYSTEM

1 tablespoon, Tbsp, T	=	3 teaspoons, tsp
1 cup, c	=	16 tablespoons, Tbsp, T
1 pint, pt	=	2 cups, c
1 quart, qt	=	2 pints, pts
1 gallon, gal	=	4 quarts, qts

WEIGHT - ENGLISH SYSTEM

1 pound, lb	=	16 ounces, oz
1 hundredweight, cwt	=	100 pounds, lbs
1 ton	=	20 hundredweight, cwt
1 ton	=	2000 pounds, lbs

LENGTH - METRIC SYSTEM

1 meter	=	1,000 millimeters
1 meter	=	100 centimeters
1 kilometer	=	1,000 meters
1 centimeter	=	10 millimeters

VOLUME - METRIC SYSTEM

1 liter	=	1,000 milliliters
1 liter	=	100 centiliters
1 kiloliter	=	1,000 liters
1 centiliter	=	10 milliliters

WEIGHT - METRIC SYSTEM

1 gram	=	1,000 milligrams
1 gram	=	100 centigrams
1 kilogram	=	1,000 grams
1 centigram	=	10 milligrams

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PERFECT SQUARES

$\sqrt{1}=1$	$\sqrt{25}=5$	$\sqrt{81}=9$	$\sqrt{169}=13$
$\sqrt{4}=2$	$\sqrt{36}=6$	$\sqrt{100}=10$	$\sqrt{196}=14$
$\sqrt{9}=3$	$\sqrt{49}=7$	$\sqrt{121}=11$	$\sqrt{225}=15$
$\sqrt{16}=4$	$\sqrt{64}=8$	$\sqrt{144}=12$	$\sqrt{256}=16$

PERFECT CUBES

$\sqrt[3]{1}=1$	$\sqrt[3]{-8}=-2$	$\sqrt[3]{64}=4$	$\sqrt[3]{-125}=-5$
$\sqrt[3]{-1}=-1$	$\sqrt[3]{27}=3$	$\sqrt[3]{-64}=-4$	$\sqrt[3]{216}=6$
$\sqrt[3]{8}=2$	$\sqrt[3]{-27}=-3$	$\sqrt[3]{125}=5$	$\sqrt[3]{-216}=-6$

$$P = \frac{\text{the number of elements in the set containing favorable outcomes}}{\text{the number of elements in the sample set}}$$

$$Midpoint = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

FORMULAS

Surface Area	Area	Volume	Perimeter/ Circumference	Others
Sphere: $A_s = 4\pi r^2$	**Triangle:** $A = {}^1\!/_2 bh$	**Sphere:** $V = {}^4\!/_3 \pi r^3$	**Perimeter of a Rectangle:** $P = 2w + 2l$	**Pythagorean Theorem:** $c^2 = a^2 + b^2$
Cylinder: $A_s = 2\pi rh + 2\pi r^2$	**Rectangle:** $A = l \times w$	**Cone:** $V = {}^1\!/_3 \pi r^2 h$	**Circumference of a Circle:** $C = 2\pi r$ or $C = \pi d$	**Convert to Fahrenheit:** $F = {}^9\!/_5 C + 32$
Cube: $A_s = 6s^2$	**Circle:** $A = \pi r^2$	**Rectangular Prism:** $V = l \times w \times h$		**Convert to Celsius:** $C = {}^5\!/_9 (F - 32)$
Lateral Surface Area of a Cone: $A_{ls} = \pi rl$	**Trapezoid:** $A = {}^1\!/_2 (b_1 + b_2)h$	**Square Pyramid:** $V = ({}^1\!/_3) lwh$		**Distance:** $d = r \bullet t$
		Triangular Prism: $V = {}^1\!/_2 bhl$		**Slope:** $y = mx = b$
		Cylinder: $V = \pi r^2 h$		**Simple Interest:** $I = prt$

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Mathematics Charts

TIME CONVERSIONS

1 week = 7 days
1 year = 12 months

CONVERSIONS

English to Metric	
1 ounce (oz)	28.35 grams (g)
1 pound (lb)	454 grams (g)
1 ton (T)	907.2 kilograms (kg)

Metric to English	
1 gram (g)	.035 ounce (oz)
1 kilogram (kg)	2.2 pounds (lb)
1 metric ton (mt)	2200 pounds (lb)

English to Metric	
1 cubic inch	16.39 cubic cm
1 gallon	3.785 liters
1 cubic foot	28.316 liters
1 quart	.946 liters

Metric to English	
1 cubic cm	.061 cubic inch
1 cubic meter	35.315 cubic feet
1 liter	1.05 quarts
1 liter	.264 gallon

English to Metric	
1 inch	2.54 cm
1 foot	0.3048 meter
1 yard	0.9144 meter
1 mile	1.6 km

Metric to English	
1 centimeter	0.3937 in.
1 meter	39.37 in.
1 meter	1.09 yds
1 kilometer	0.6213 mile

AREA CONVERSIONS

1 square centimeter = 0.155000 sq in.
1 square meter = 10.763910 sq ft
1 square inch = 645.16 sq mm
1 square foot = 0.09290304 sq meter
1 square yard = 0.836127 sq meter
1 square foot = 144 sq in.

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